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Høgskolen i Østfold
Arbeidsrapport 2010:4

Online-versjon (pdf)

Utgivelsessted: Halden

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Høgskolen i Østfold. Arbeidsrapport 2010:4
© Forfatteren/Høgskolen i Østfold
ISBN: 978-82-7825-325-0
ISSN: 1503-6677

Housing careers, P/R-ratios and rental equivalence

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Abstract:
This paper analyses price-rent ratios under different housing career structures. A housing market with three segments for owner-occupation and a segment with rental housing is applied while a housing career is characterised by how households move between rentals and owner-occupation. While rents are completely passed-through to aggregate house prices when rentals represent alternative housing to all forms of owner-occupation, more realistic housing career structures induce strict conditions for complete pass-through. In a heterogeneous housing market with equity induced up-trading complete pass-through is conditional on the ratio between market segments only indirectly affected by rent. This ratio is determined by the distribution of equity effects between market segments and the price elasticity of the different segments, and is therefore context specific. Stated differently, in most housing markets rents will not be completely passed through to house prices in the short-run.

1. Introduction

Economic theory predicts that the price of an asset should equal the present value of its future earnings. If not so, arbitrage opportunities arise. In a close to efficient market any discrepancy between price and earnings tends to be rare and short lived. Leamer (2002) argued the price-rent (PR) ratio to be for the housing market what the price-dividend ratio is for the stock market. The housing market is regarded as markets where transactions costs, such as cost of information and cost of purchase, create market frictions, and lower the opportunity for arbitrage seeking agents to restore market efficiency (Hilbers et al, 2008). Hence, such a market would display time persistent deviations in the PR-ratio from its long-term trend.

In the housing market PR-ratios are keenly watched and used for a variety of purposes. They are used as indicators for potential housing market bubbles (Himmelberger et al, 2005), and as a predictor for market corrections. The latter because the PR- ratios are known to be mean reverting (Gallin, 2008). Various other aspects of the relationship between rents and house prices are analysed by, amongst others, Case and Schiller (1990), Meese and Wallace (1994), and Campbell et al (2006). Equally important, many countries rely on imputed rents when calculating housing consumption for owner-occupied housing. This approach is known as “rental equivalence”, and relies on the assumption that rents mirror the cost of owner-occupied housing (Bourassa, 1998). But still, if rents do not mirror house prices - and vice versa – developments in the consumer price index and ultimately inflation targeting misses out on the cost of owner-occupation.

Our point of departure is the short term dynamics of PR-ratios. If ownership and rental were perfect substitutes, any shift in rent would (in the absence of transaction costs) more or less immediately be balanced by corresponding shift in house prices. If so, rental equivalence and strategies for assessing housing market equilibriums on the basis of PR-ratios would be efficient. In practice however, there is not one housing market and one rental market. Housing markets at a finer level reflect our housing consumption profile over the life cycle. As a young adult most of us live in small rented dwellings. As we age, marry and have kids, we seek larger dwellings, and to a greater extent owner-occupied housing.

This kind of housing ladder implies that housing market structures tend to display the following regularity; a market segment for starter homes where owner-occupied housing coexist with a considerable rental market while market segments for intermediate and family homes are dominated by owner occupied dwellings, as rental alternatives are more or less absent in these market segments.

Such a market structure has ramifications for the substitutability between renting and owning. The up-trading structure implies that renting is an imperfect substitute for owner-occupation, as it only represents an alternative to owner-occupation at the first step of the housing ladder. Hence, it is only the market segment

prices on the first step of the housing ladder which *directly* is affected by changes in rent. Further up the housing ladder are market segment prices only *indirectly* affected by changes in rent.

Second, capital gains from existing home-ownership are important for the ability to trade up the housing ladder. Lately, equity gains from home-ownership have been addressed by a number of papers, see for instance Kajuth (2010).

The basic question addressed in this paper is whether a heterogeneous market structure where segments are linked through equity induced up-trading affects the PR-ratio in the short-run?

We answer yes, and illustrate how a shock in the rental market, say induced by a shift in labour supply or demand, affects the short-term PR-ratio in the absence of transaction costs. If the submarket of starter homes is the only one that contains a rental alternative, an increase in rent is only indirectly passed-through to the market segment prices further up the housing ladder. This creates a short-term downward bias in the PR-ratio compared to the long-run equilibrium. Our multimarket equilibrium model displays how direct and indirect effects impact the PR-ratio, and provide insight regarding how different housing careers are likely to have a bearing on short-term deviations in the PR-ratio. A particular emphasis is placed on how the distribution of equity effects between market segments impacts the rent pass-through.

The contribution of this paper relates to those of Ortalo-Magne' and Rady (1999, 2006), and Mankiw et al (1989) highlighting the importance of the housing demand of young households and the first steps of the housing ladder for aggregate housing market fluctuations. Our approach also relates to the argument of Bourassa et al (2003) on the importance of different market segments by integrating a house price index that *directly* distinguishes between market segments with different housing career structures. In the case of up-trading this allows market segments an *indirect* effect on the house price index depending on their positions on the housing ladder. The model is applied to analyse how rent is passed-through to the house price index in the short run and discuss the conditions for complete pass-through under different housing career structures. Hence, the paper takes into account the criticism of Cameron et al (2006) in the sense that it relates the PR-ratio to a model for housing demand. The model provides theoretical support for the findings of Ayuso and Restoy (2006), Krainer and Wei (2004) and Gallin (2008) that changes in PR-ratios are driven by changes in expected future house price growth rather than rents and that despite their long-run cointegration the short-run effect on house prices from changes in rent is weak. The short-run deviation in the PR-ratio from its long-run equilibrium ratio questions the argument of Favilukus et al (2010) on how financial innovations that reduce transaction costs in the housing markets also will reduce short-term deviations in PR-ratios.

The paper is structured as follows: The different housing career structures and the supply side simplifications are given in section (2). In section (3) the expressions for housing demand in the different housing career structures are specified. In addition, market segment prices, house price indices and rent pass-through are derived. The last part concludes.

2. A housing market with owner-occupation and rental housing

We consider a simplified version of a housing market where owner-occupation can occur in three *distinct* segments, starter (s), intermediate (m), and family homes (f). Each of the market segments is directed towards three different types of households, labelled starter (s), intermediate (m) and family (f), represented by different age groups. In addition to the market segments for owner-occupation there exists a rental market. A housing career is characterised by how households move between these market segments.

In the following we consider two different housing career structures:

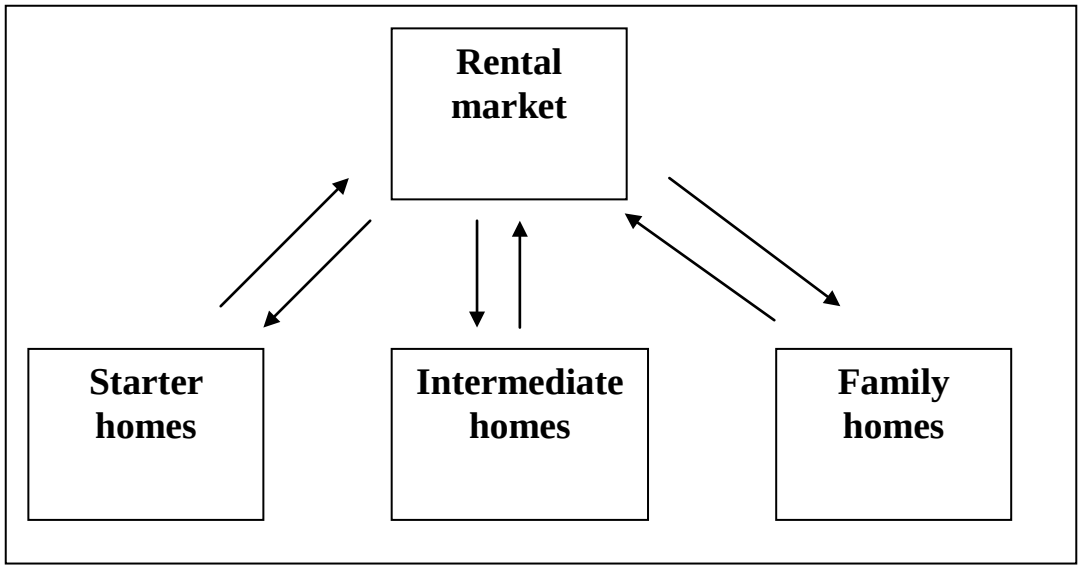
1. A housing career where rentals represent alternative housing to home ownership in all market segments for owner-occupation Rental alternatives to owner-occupation are present in all market segments and there are no links between the segments for owner-occupation.

2. A housing career where rentals only is considered alternative housing to home ownership in the market segment for starter homes, making it an imperfect substitute for owner-occupation. The market segments for owner-occupation are now linked through a housing ladder. The ladder has three steps where a starter home is the first-, an intermediate home the second-, and a family home the final step. In the presence of up-trading we consider two distinct sub-cases: one where owner-occupation in different market segments only is linked through substitution effects and one where market segments also are linked through equity effects.[1]

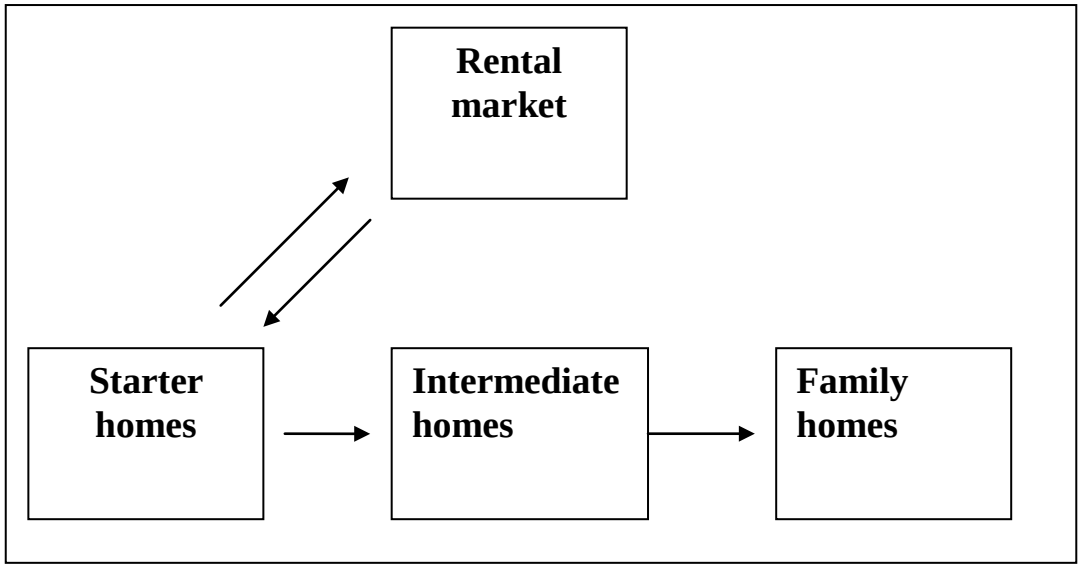
The housing career structures can be visualised as follows.

(I) Rental housing as a perfect substitute for owner-occupation and no up-trading

[1] A third alternative is a housing career which contains both up-trading and renting, and rentals serve as perfect substitutes for all types of owner-occupation. When it comes to rent pass-through this alternative only differs from our extremes with respect to scaling, determined by the share of households considering rentals as alternatives to home ownership in each market segment.



(II) Rental housing as a substitute for starter homes, and up-trading linking the market segments for owner-occupation



The motivation for discussing these two extremes is to highlight the difference between a *direct* and an *indirect* channel for how the rental market affects house prices in the short run. To highlight the short-run impact of the different housing career structures, we introduce two simplifications: First, we keep the rental market exogenous to the model. Second, in each of the three market segments for owner-occupation housing supply is fixed:

1) $$D_i = S_i \qquad i = s,m,f$$

Where, S_i, is housing supply and, D_i, is demand in market segment i. Market segment prices ensures equilibrium in each segment. The price index for the housing market as a whole is:

2) $$P = \Sigma_i \ \alpha_i P_i$$

where market segment sizes, α_i , determine the different segments direct impact on the house price index.

3. Market segment prices and house price indices under different housing careers

When considering market segment prices, the house price index and rent pass-through under different housing career structures, we start out by assuming that a household's demand for owner-occupied housing depends on the price of housing, *P*, households net equity, *E*, and the price of alternative housing. If renting is the alternative to owner-occupation rent, *R*, is the price of alternative housing, while the price of alternative housing equals, P_j, (j≠i) if given by owner-occupation in another market segment.

(I) Rental housing as a perfect substitute to all forms of owner-occupation and no up-trading:

In each of the three market segments for owner occupation housing demand equals:

3) $$D_i = k_i + r_i R - p_i P_i \qquad \text{i= s,m,f.}$$

Where, r_i , is the price elasticity of alternative housing in segment i (the indirect price elasticity), and, p_i , the direct price elasticity of segment i. Finally, k_i, is a constant facilitating our linearization of demand and incorporate amongst other factors the income effect in demand.

For illustrative purposes, let us for the first case include the expressions for market segment prices and the house price index. From (1) market segment prices can be expressed as:

4) $$P_i = \frac{1}{p_i}\left[k_i + r_i R - S_i\right] \;\; \forall \text{ i}$$

Each market segment price is determined by demand and supply in its own market segment. Inserting for market segment prices in expression (2) gives the house price index as

5) $$P = \sum_i \alpha_i \frac{k_i}{p_i} + \sum_i \alpha_i \frac{r_i}{p_i} R - \sum_i \alpha_i \frac{1}{p_i} S_i$$

As no up-trading occur supply and demand in each market segment has a direct impact on the house price index according to the segment size. If, for instance, the direct price elasticity is equal in all market segments, and the segments are of equal size, a supply increase has the same negative impact on the house price index irrespective of in which segment the increase occurs.

When considering how rent, *R*, is passed-through to the house price index, we differentiate the house price index with respect to rent as an indicator of *rent pass-through*.

6) $$\frac{dP}{dR} = \alpha_s \frac{r_s}{p_s} + \alpha_m \frac{r_m}{p_m} + \alpha_f \frac{r_f}{p_f}$$

Since $\alpha_s + \alpha_m + \alpha_f = 1$, a sufficient set of conditions for complete pass-through, $\frac{dP}{dR} = 1$, is $r_s = p_s$, $r_m = p_m$, $r_f = p_f$. This complete pass-through condition is referred to as an *elasticity constraint.*

(II) Rental housing as a perfect substitute for starter homes only, and up-trading linking the market segments for owner-occupation

Now we consider rent pass-through in a housing market where renting is considered an imperfect substitute to owner-occupation and rentals only represent alternative housing for starter homes. First, we ignore equity effects and relate segments for owner-occupation through a conventional substitution effect.

a) Up-trading and substitution effects.

The demand for owner-occupation of starter-, intermediate- and family homes can now be expressed as:

7) $D_s = k_s + r_s R - p_s P_s$

8) $D_m = k_m + r_m P_s - p_m P_m$

9) $D_f = k_f + r_f P_m - p_f P_f$

When a housing careers is as illustrated in figure 1, the rental market represents alternative housing for starter homes, while starter (intermediate) homes is alternative housing for intermediate (family) homes. The relevant market segment prices and the house price index are now given in the appendix.

In contrast to when rentals represent alternative housing for owner-occupation in all market segments, up-trading relates rent either directly or indirectly to the market segment prices. While the price of starter homes only is affected by supply and demand in the starter home segment, the indirect effect accompanying substitution also relates prices further up the ladder to the supply and demand conditions in the market segment for starter homes. For instance, the price of intermediate homes is *directly* related to supply and demand in the market segment for intermediate homes, but through substitution also *indirectly* to the supply and demand for starter homes. And - being on the top of the housing ladder - the price of family homes is *directly* affected by the market conditions in the segment for family homes, but through substitution also *indirectly* to supply and demand in both the segment for starter and in the segment for intermediate homes. When it comes to rent, it has a *direct* effect on the price of starter homes, but through substitution also *indirectly* on both the price of intermediate and on the price of family homes.

Rent pass-through now equals:

10) $$\frac{dP}{dR} = \frac{r_S}{p_S}\left[\alpha_s + \frac{r_M}{p_M}\left[\alpha_m + \frac{r_F}{p_F}\alpha_f\right]\right]$$

Since, $\alpha_s + \alpha_m + \alpha_f = 1$, equality between the direct and the indirect price elasticity in all market segments, i.e., $r_s = p_s$, $r_m = p_m$, $r_f = p_f$, is again a sufficient condition for complete pass-through. Even when rentals

are not alternative housing to all types of owner-occupation, and the segments for owner-occupation are linked through substitution, is the elasticity constraint sufficient for complete pass-through.

b) Owner-occupied housing and equity effects

In order to allow households to take advantage of capital gains from existing homeownership when climbing the housing ladder we introduce housing equity. For instance, aggregate equity enters the demand for intermediate houses as an increasing function of the price for starter homes: $E_m = E_m(P_s)$. Likewise, the equity effect on the demand for family homes is positively related to the price of intermediate homes, $E_f = E_f(P_m)$. For simplicity we consider linearized versions of these equity functions:

11) $$e_m E_m = e_{mo} E_{0m} + e_{ms} P_s$$

12) $$e_f E_f = e_{fo} E_{0f} + e_{fm} P_m$$

Where, e_i , represents the equity elasticity of owner-occupation in market segment *i*, and, e_{ms} (e_{fm}), measures how strongly housing equity of starter (intermediate) homes impact on the up-trading to intermediate (family) homes. E_{0m} and E_{0f} are exogenous equity components.

The expressions for housing demand including the equity effects are given in the appendix, along with the expressions for the relevant market segment prices and the house price index. Market segment prices can again be influenced by rents either directly or indirectly, where the latter effect now contains both substitution and equity components.

Rent pass-through equals:

13) $$\frac{dP}{dR} = \frac{r_S}{p_S}\left[\alpha_s + \alpha_m\left[\frac{e_{ms} + r_{ms}}{p_m}\right] + \alpha_f\left[\frac{e_{fm} + r_{fm}}{p_f} + \frac{e_{ms} + r_{ms}}{p_m}\right]\right]$$

When applying the same set of conditions as above, $\alpha_s + \alpha_m + \alpha_f = 1$, and the elasticity constraint, $r_s = p_s$, $r_{ms} = p_m$, $r_{fm} = p_f$, the condition for complete pass-through, $\frac{dP}{dR} = 1$, reduces to

14) $$\frac{\alpha_m}{\alpha_f} = \frac{e_{fm}}{p_f}\left[1 + \frac{p_m}{e_{ms}}\right] + 1$$

When equity influence on up-trading and rentals only are considered substitutes for starter homes, complete pass-through is in addition to the conditions above depending on the housing market structure, represented by the ratio between the market segments only indirectly affected by rent. The market structure that ensures

complete pass-through is determined by the direct price elasticises and the distribution of equity effects between these two segments and is therefore context specific.

In order for pass-through to be complete the housing market structure has to ensure that the indirect effects accompanying the equity effects are strong (weak) enough to compensate for the lack of direct links to the rental market. Equation (14) shows that in the absence of equity effects from intermediate to the family homes, i.e. $e_{fm} = 0$, complete pass-through is conditional on equality between the market segments for intermediate and family homes. When such equity effects are present, $e_{fm} > 0$, the segment for intermediate homes has to exceed that of family homes. This is in order to constrain the indirect effect by making the ballpark of it only "climb one step" on the housing ladder. Likewise, as the direct price elasticity for family homes approaches zero the size of the segment for intermediate homes convergences towards that of family homes, as the latter's contribution to how rent is passed through to the house price index decreases. Vice versa, if the price elasticity of intermediate homes equals zero, this segment should exceed that of family homes.

Comparative statics shows the asymmetric effect of both equity and price elasticises on the complete pass-through condition. First of all, while the ratio of intermediate to family homes is positively related to the equity effect from intermediate to family homes, e_{fm}, it is negatively related to the equity effect from starter to intermediate homes, e_{ms}. Second, while the ratio increases with the price elasticity of intermediate homes, p_m, it decreases with the elasticity for family homes, p_f. Hence, equity induced up-trading complicates the equilibrium assessments of housing markets.

6. Conclusions

Housing careers can take different forms. For some households renting is always an alternative to owner-occupied housing. For others it is only an alternative in the beginning of adulthood, often before being able to fulfil down-payment conditions given by mortgagees. Hence, real housing markets contain market segment specific distributions between rentals and home ownership. Still, housing market equilibriums are often assessed by PR-rent ratios at the aggregate level, where renting implicitly represents alternative housing for all forms of owner occupied housing.

This paper argues that housing career structures matter for how rent is passed-through to the aggregate house price index, and that the degree of pass-through varies according to the distribution of equity effects between market segments. When rentals represent alternative housing for all forms of owner-occupation and no up-trading occurs, complete pass-through is related to an elasticity constraint. Even when rentals do not represent alternative housing for all types of owner-occupation and the market segments for owner-occupation are linked through substitution, complete pass-through is conditional on the same constraint,

perfect substitutability between rentals and owner-occupied housing. When the capital gains of home ownership are ignored, rentals and owner-occupation supply households with the same possibilities for consumption of housing services, and substitutability might be argued to be perfect.

However, when capital gains and equity effects are allowed to impact on up-trading complete pass-through is less obvious, and conditional on the housing market structure. Complete pass-through is now related to the ratio between market segments only indirectly affected by rent. This ratio has to ensure that the indirect effects from changes in rent through substitution and equity induced up trading are strong (weak) enough to ensure complete pass-through. This ratio is again determined by the distribution of equity effects between the different steps of the housing ladder.

In a heterogeneous housing market where up-trading between market segments is equity induced the market structure necessary for complete pass-through is context specific. It might both change over time and vary between countries. Housing markets continuously fulfilling the complete pass-through constraint is now obviously unrealistic. Stated differently, in the presence of equity induced up-trading short-term deviations from long-term trends in the PR- ratio should be expected.

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Appendix

Market segment prices and the house price index under different housing careers

(ii) *Owner-occupied housing and substitution effects*

By applying the equilibrium conditions in (1), and inserting for housing demand from (7) - (9), market segment prices can be expressed as:

A1) $$P_s = \frac{1}{p_s}\left[k_s + r_s R - S_s\right]$$

A2) $$P_m = \frac{1}{p_m}\left[k_m - S_m + \frac{r_m}{p_s}\left[k_s + r_s R - S_s\right]\right]$$

A3) $$P_f = \frac{1}{p_f}\left[k_f - S_f + \frac{r_f}{p_m}\left[k_m - S_m + \frac{r_m}{p_s}\left[k_s + r_s R - S_s\right]\right]\right]$$

The house price index equals:

A4) $$P = \frac{\alpha_f}{p_f}\left[A_f\right] + \left[\frac{\alpha_m}{p_m} + \frac{\alpha_f r_f}{p_f p_m}\right]\left[A_m\right] + \left[\frac{\alpha_s}{p_s} + \frac{\alpha_m r_m}{p_m p_s} + \frac{\alpha_f r_f r_m}{p_f p_m p_s}\right]\left[A_s\right]$$

where $A_f = \left[k_f - S_f\right]$, $A_m = \left[k_m - S_m\right]$ and $A_s = \left[k_s + r_s R - S_s\right]$.

c) Owner-occupied housing and equity effects

By inserting the equity functions in (11) and (12) into (8) - (9) housing demand in each market segment can be expressed as:

A5) $$D_s = k_s + e_s E_s + r_s R_s - p_s P_s$$

A6) $$D_m = k_m + e_{mo} E_{0m} + e_{ms} P_s + r_{ms} P_s - p_m P_m$$

A7) $$D_f = k_f + e_{fo} E_{0f} + e_{fm} P_m + r_{fm} P_m - p_f P_f$$

where E_s is an exogenous variable. Combed with the equilibrium conditions in (1), market segment prices can be derived as:

A8) $$P_s = \frac{1}{p_s}\left[c_s + e_s E_S + r_s R - S_s\right]$$

A9) $$P_m = \frac{1}{p_m}\left[c_m - S_m + \frac{e_{ms} + r_{ms}}{p_s}\left[e_s E_s + r_s R - S_s\right]\right]$$

A10) $$P_f = \frac{1}{p_f}\left[c_f - S_f + \frac{e_{fm} + r_{fm}}{p_m}\left[c_m + e_s E_s - S_m + \frac{e_{ms} + r_{ms}}{p_s}\left[c_s + r_s R - S_s\right]\right]\right]$$

where $c_s = k_s$ $c_m = k_m + e_m E_{0m} + \frac{e_{ms} + r_{ms}}{p_s} c_s$ and $c_f = k_f + e_f E_{0f} + \frac{e_{fm} + r_{fm}}{p_m} c_m$.

The house price index equals:

A11) $$P = \frac{\alpha_f}{p_f}\left[A_f\right] + \left[\frac{\alpha_m}{p_m} + \frac{\alpha_f}{p_f}\left[\frac{e_{fm} + r_{fm}}{p_m}\right]\right]\left[A_m\right] + \left[\frac{\alpha_s}{p_s} + \frac{\alpha_m}{p_m}\left[\frac{e_{ms} + r_{ms}}{p_s}\right] + \frac{\alpha_f}{p_f}\left[\frac{e_{fm} + r_{fm}}{p_m}\frac{e_{ms} + r_{ms}}{p_s}\right]\right]\left[A_s\right]$$

where A_i are the same as above.